

Uncertainty of Geometrical Product Specification Measurements

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Abstract

Usually, the measured points are taken for the extracted feature of a surface and to calculate the associated features e.g. the total least square or adjacent straight lines. However, it may be shown by simulations, that this model is not suitable to evaluate the uncertainties of the geometrical properties of surfaces. The alternative is an iteratively filtered mean profile with known uncertainty, to which any geometrical features may be associated.

1. Introduction

In GPS measurements two fundamental problems have to be solved:

1. There is a fundamental contradiction between the definitions of the tolerances of form, direction, runout and the adjacent geometrical features [1] as well as the definition of the stated uncertainty according to the GUM [2]: the tolerance zone has to cover the extreme points of the surface, respectively, and the adjacent features to these points are calculated. The stated uncertainty according to the GUM refers to the best estimate of the value of the measurand represented by the mean value. The uncertainty of extreme values is not part of the GUM.
2. The real surface is measured by a limited number of points. Generally, the extreme points of the surface may not be known measuring discrete points, but only the extreme measured points as a more or less good approximation. The stated uncertainty has to take into account this influence, too.

The central question in geometrical product specification (GPS) measurements is with reference to the geometric features. The standard ISO 14660 defines the terms "real", "extracted" and "associated feature" [3], [4]. The extracted feature is an approximated representation of the real feature, obtained by extracting a finite number of points from the real feature. This extraction is performed in accordance with specified conventions. One such convention is the use of standardized Gaussian wavelength filters with a stated cut-off wavelength, in form measurements e.g. $\lambda_c=0.8$ mm [5]. The problems caused by this kind of filtering are discussed in [6].

The solution was found in a modification of the filtering process. By iterative variation of the cut-off wavelength, a filter may be found of which the residual errors are completely uncorrelated und independent. These conditions may be proven by statistical tests. The result is an extracted feature, e.g. a mean profile of the surface, with a well-known uncertainty, containing the uncertainty contribution of local deviations of form measured with a limited number of points, and the dispersion of the measuring system. This extracted feature may be used to associate total least square or adjacent features, corresponding to the various demands on the functional properties of the surfaces [6]. The attempt to prove these proposals results in the question of demonstrability of scientific theories in general.

2. Models and evaluation method

Usually, in technical sciences theories are proven by practical experiments. In this case various methods of evaluation of the uncertainty for various definitions of extracted and associated features have to be tested. Because of the uncertainty describing the dispersion of the values that could reasonably be attributed to the measurand, the correctness of the evaluation has to be surveyed. This may be done by a lot of repeated measurements and calculating the share of correct respective non-correct uncertainty evaluations. According to the GUM, for a confidence level of 95%, the evaluated uncertainty may be increased by the actual measured deviation in 5% of the cases on average.

Executing real measurements would take a fair amount of time, taking into account the measuring of equipment as well as the stabilization of environmental conditions. Therefore, the results could be falsified by random effects. That's why in techniques are preferred computer simulations. Thus all relevant quantities of influence may be purposefully varied to test their effects on the measurand.

However, the results of computer simulations (just like every other method of proving scientific theories) must also be critically viewed. A test result in accordance with the thesis alone does not completely prove the theory in every case. It depends on the kind of thesis, as it was discussed by the philosopher Karl R. Popper 70 years ago [7]. Following his argumentation, we have to distinguish between "all-sentences" and universal "there is-sentences".

An all-sentence is a thesis like e.g. "All ravens are black". This is a positive statement, and it cannot at all be proven, because nobody can know and see *all* ravens in the world. But it could be refuted by any witness who saw anywhere in the world a white (or even a differently coloured) raven. A universal "there is-"sentence is a thesis like e.g. "There are white ravens". This is a negative statement, and it may easily be proven as described above, but not disproved. Applied to science, any number of well-designed experiments is not able to prove a theory. But only one experiment creating a result which is not according to the theory would completely disprove it.

Referring to the above described fundamental problems of GPS measurements, the influence quantities during the simulated experiments should be varied until at least one (or both) theories are refuted. The remaining theory should be used to evaluate uncertainties of the deviations of form, direction, runout and of the adjacent geometrical features, until perhaps a better one can be found.

3. Design of the simulated measurements

The computer simulations are carried out on the sample of a simple surface profile. The original profile consists of a straight line with defined systematic local deviations of form (figure 1a). They are superposed by more or less large random deviations with various probability distributions, e.g. rectangular (figure 1b), triangular or normal (figure 1c). Simulating the measurements, the described original profile will be superposed by more or less large random deviations with various probability

distributions, e.g. rectangular, triangular or normal with different numbers of measuring points.

The systematic deviations of the measuring system as well as other influence quantities, like e.g. temperature, are not taken into account because they do not cast doubt on the evaluation of the uncertainty. Calculating the results, the following extracted and associated features are used:

- Measured points (figure 2)
- Adjacent straight line to the measured points (figure 3)
- Total least square line to the measured points (figure 4)
- Iteratively filtered mean profile (figure 5)
- Adjacent straight line to the mean profile (figure 6)
- Total least square line to the mean profile (figure 7)

Analysing the simulated measurements, the following geometrical properties of the measured surface profiles are calculated:

- Deviations of form, referring to the adjacent and the median straight lines
- Coordinate of the centre of the profile
- Profile points

The uncertainties of the mean profile and its geometrical properties are evaluated for every single sample according to [6] and [8] using a spline filter. The difference to the Gaussian filter is negligible, and it takes much less calculation time. The uncertainties of the geometrical properties calculated from the measured points are evaluated using the standard uncertainty of the random and independent deviations as described in [6]. Multiplying the standard uncertainty with a coverage factor of $k=2$ results in the expanded uncertainty.

The test criterion is the number of cases in which the expanded uncertainty of measurement is exceeded by the deviations of the geometrical properties. For a confidence level of 95% according to the GUM the evaluated uncertainty may be increased by the actual measured deviation in 5% of the cases on average. However, because of random effects, the actual share of a simulation may be smaller or greater than this value. The limits of this dispersion of the values may be characterized by the confidence interval of the Poisson distribution, expressing the number of cases to find a failure within the sample. The theory would be disproved, if the upper limit of the confidence interval of any geometrical property were exceeded.

In this case, usually 2.000 simulations are executed. For a stated level of confidence of 95% in 100 of these simulations, the expanded uncertainty may be exceeded without disproving the theory. The confidence limits of the Poisson distribution for the same level of confidence are 81,4 (4.1%) and 121,6 (6.1%) [9]. Not until the deviations of at least 122 simulations out of 2.000 are larger than the uncertainty is the theory disproved.

4. Results

The original profile (figure 1) is a sinusoidal function with an amplitude of $a=1$, superposed by normally distributed random deviations with standard deviations of $s_0=1, 0.3$ and 3 . The samples are represented by superposed, normally distributed random deviations with standard deviations of $s_s=1, 0.3$ and 3 . The simulations are carried out with various numbers of points within the samples. Examples of the results are shown in table 1.

Table 1: Standard deviations and share of simulated samples with $n=10$ points with deviations exceeding the stated uncertainty of the samples (in %); deviation of form (above), coordinate of the centre of the profile (centre), and profile points (below)

Standard deviation of the profile s_0	0.3			1			3		
	0.3	1	3	0.3	1	3	0.3	1	3
Standard deviation of the sample s_s	0.3	1	3	0.3	1	3	0.3	1	3
1. Profile points and measured points of the sample (figure 2)	13.5 14.5 2.3	0.6 2.6 0.7	1.5 0.3 0.3	19.0 25.2 3.0	5.8 11.2 1.6	0.3 0.8 0.5	20.0 15.0 3.3	19.9 13.6 3.0	4.5 4.6 1.4
2. Adjacent straight lines to the profile points (figure 3)	13.5 18.1 12.6	0.6 1.2 1.0	1.5 1.1 0.9	19.0 30.2 20.5	5.8 11.3 8.7	0.3 0.6 0.8	20.0 29.7 23.0	19.9 24.2 18.7	4.5 7.4 6.7
3. Total least square lines to the profile points (figure 4)	4.1 2.2 1.9	0.3 3.3 3.1	1.0 3.7 3.7	5.4 0.8 1.5	1.6 2.6 2.3	0.1 4.0 3.6	6.8 1.3 1.7	4.7 2.0 2.4	0.7 3.1 3.1
4. Iteratively filtered mean profiles (figure 5)	1.2 0.6 0.6	1.5 2.2 2.3	1.6 1.4 2.2	0 0 0	0.1 0 0	0.9 0.3 0.5	0 0.8 1.2	0 0.9 1.4	0.1 0.9 1.7
5. Adjacent straight lines to the mean profiles (figure 6)	1.2 0 0.1	1.5 0.3 0.3	1.6 0.4 0.4	0 0 0	0.1 0.1 0	0.9 0.3 0.3	0 0 0	0 0 0	0.1 0.1 0
6. Total least square lines to the mean profiles (figure 7)	0.5 2.0 1.5	1.5 2.6 2.6	2.4 2.7 3.0	0 0.3 1.1	0.1 2.0 1.8	0.9 3.1 3.0	2.3 0.4 1.0	2.6 1.5 1.8	2.7 2.4 2.5

Using the points of the original profile and the measured points of the samples, the results strongly depend on the standard deviations of the original profile and of the samples. In most cases the deviations are bigger than the stated uncertainties from the samples. An exception is the total least square line to the measured points (3.). With the exception of this case, the measured points are no appropriate model to describe the deviations of the surface. It is disproved by the simulations.

Using the iteratively filtered mean profiles, all deviations are significantly smaller than the stated uncertainties from the samples. The uncertainty is evaluated as a bit too large, but not too small. The mean profile and the total least square, respectively, as well as the adjacent straight lines associated with it are an appropriate model to describe the deviations of the surface. It is not be disproved by the simulations.

5. Summary

Using the points of the original profile and the measured points of the samples, the uncertainty of the geometrical properties of technical surfaces may not be evaluated correctly. An exception is the total least square straight line to the measured points, but it is not appropriate to characterize all functional demands of the surface.

Thus, the iteratively filtered profile is presently the best known model for the extracted feature according to ISO 14660. At every point of the mean profile the uncertainty may be evaluated, and the total least square as well as the adjacent straight lines may be associated with it together with their uncertainties.

6. References

- [1] ISO 1101 (1983): Technical drawings, geometrical tolerancing; tolerances of form, orientation, location and runout; generalities, definitions, symbols on drawings
- [2] Guide to the Expression of Uncertainty in Measurement (GUM). International Standardization Organization, Geneva 1993
- [3] ISO 14660-1 (1999): Geometrical Product Specifications (GPS) - Geometrical Features - Part 1: General terms and definitions
- [4] ISO 14660-2: (1999): Geometrical Product Specifications (GPS) - Geometrical Features - Part 2: Extracted median line of a cylinder and a cone, extracted median surface, local size of an extracted feature
- [5] ISO 11562 (1996): Geometrical Product Specifications (GPS) - Surface texture: Profile method - Metrological characteristics of phase correct filters
- [6] Hernla, M.: Inspection of surface geometry. X. International Colloquium on Surfaces, Shaker Verlag Aachen 2000, pp. 23-32
- [7] Popper, Karl R.: Logik der Forschung (Logik of research). J. C. B. Mohr (Paul Siebeck), Tübingen 1982
- [8] Hernla, M.: Anwendung von Filtern bei der Auswertung gemessener Oberflächenprofile (Application of filters in evaluation of measured surface profiles). *tm Technisches Messen*, München, 67 (2000) 3, S. 128-135
- [9] DIN 53804 Teil 2: Statistische Auswertungen; Zählbare (diskrete) Merkmale. (Statistical analysis. Countable discrete characteristics) Beuth Verlag Berlin 1985

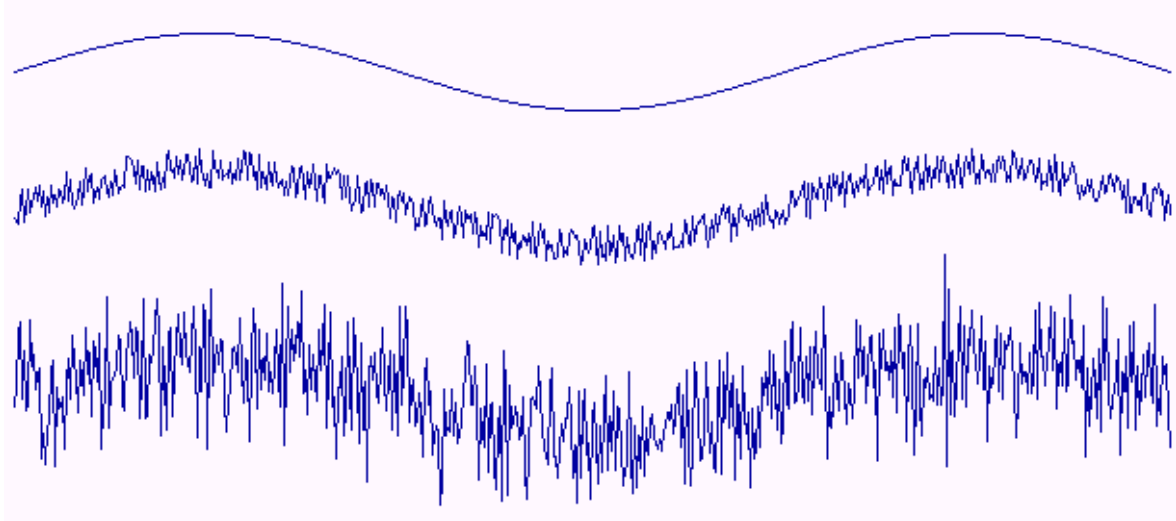


Figure 1: Original profile, consisting of a straight line with systematic local deviations of form (above), superposed by random deviations with rectangular distribution (middle), respectively by random deviations with normal distribution (below)

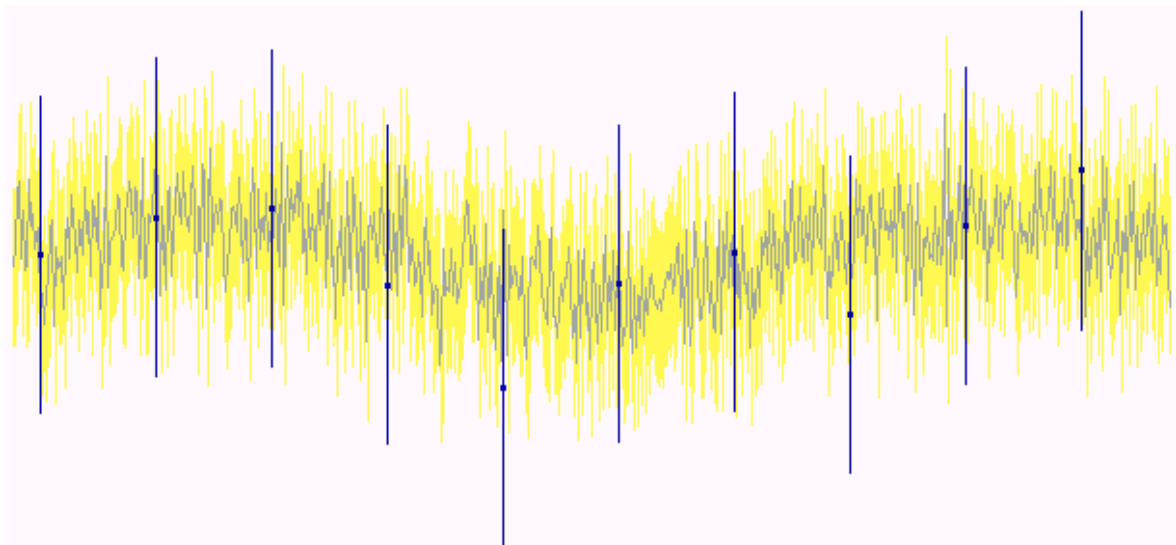


Figure 2: Original profile from figure 1 (below) and measured points of the sample with random dispersion intervals of the original profile (light grey lines) and of the sample (black lines)

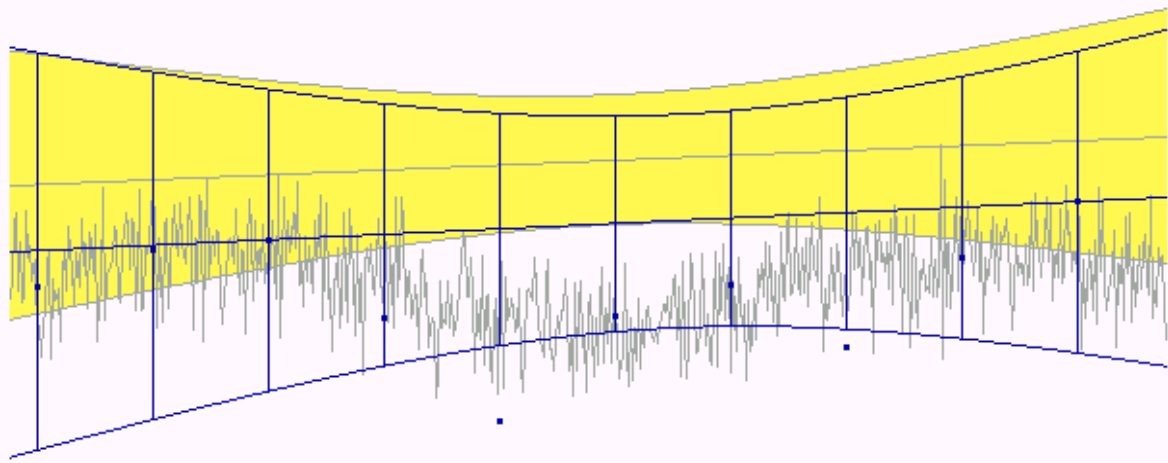


Figure 3: Original profile (grey) and measured points of the sample (black) from figure 2 with adjacent straight lines and confidence intervals

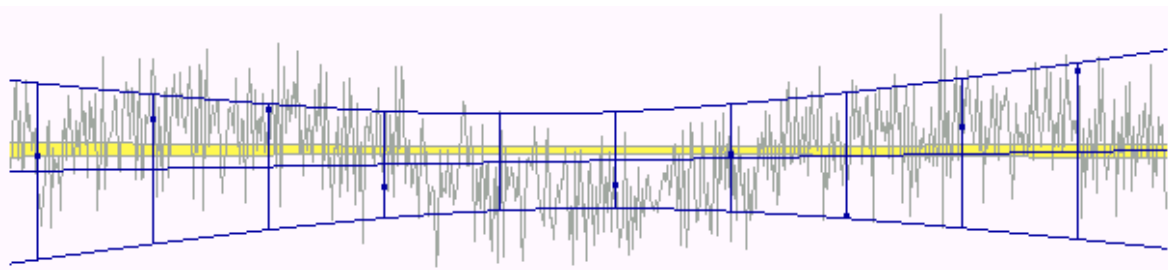


Figure 4: Original profile (grey) and measured points of the sample (black) from figure 2 with total least square lines and confidence intervals

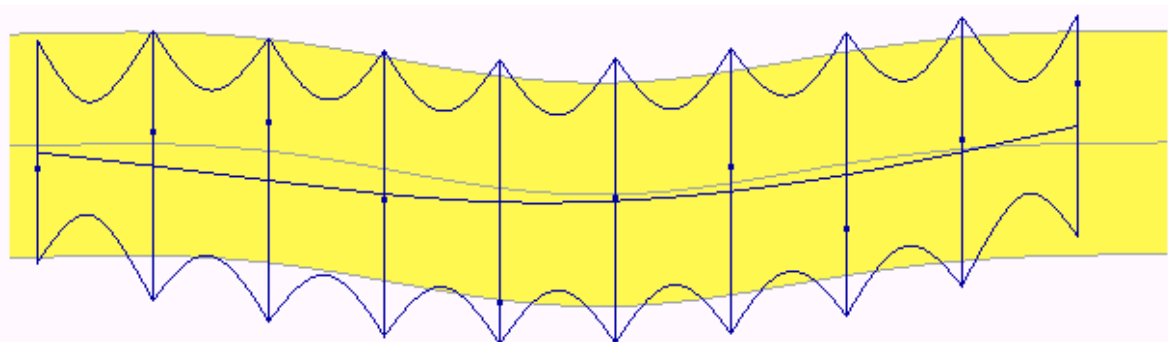


Figure 5: Iteratively filtered mean original profile from figure 2 (grey), measured points and iteratively filtered mean profile of the sample (black) and confidence intervals

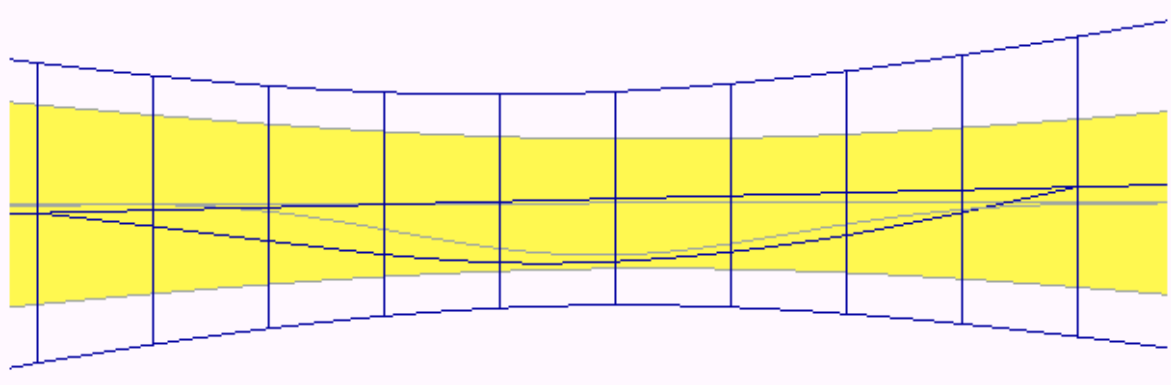


Figure 6: Mean profiles from figure 5 with adjacent straight lines to the mean profiles and confidence intervals

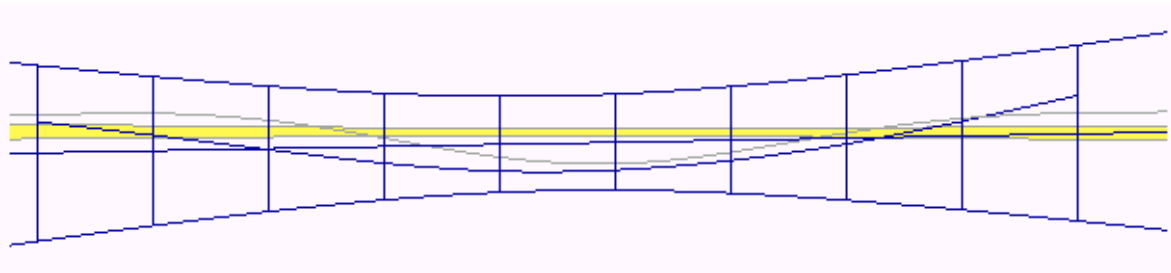


Figure 7: Mean profiles from figure 5 with total least square lines to the mean profiles and confidence intervals